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COMMENT

A comment on 'The classical limit for the Holstein–Primakoff representation in the soliton theory of Heisenberg chains'

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Abstract. Škrinjar and co-workers used a Holstein–Primakoff operator method to obtain a classical Hamiltonian. It is shown that the same Hamiltonian can be derived directly via averaging over spin coherent states, so the former contains no extra information in the limit $s \to \infty$.

In the paper by Škrinjar *et al* [1] a classical Hamiltonian was derived via the following procedure: bosonisation of the initial quantum Hamiltonian through Holstein–Primakoff (HP) operator transformations followed by averaging over the Glauber coherent states. Performing this procedure, the authors dropped all the terms arising due to operators a and a^+ ordering (the so-called 'quantum' corrections). The thus-constructed Hamiltonian is a classical analogue of the quantum one for $S \to \infty$ and can be simply obtained through substituting Bose operators by C-numbers.

The same classical Hamiltonian can be obtained directly via averaging over spin coherent states.

(i) The statement that both the HP and spin coherent states (SCS) approaches in the classical limit, $s \to \infty$, give rise to the same result (if one uses the total HP series) is quite trivial since (see, e.g., [2], equation (11)) in this limit Glauber CS coincide with the SCS up to a reparametrisation. The Hamiltonian is given respectively in terms of either $|\alpha|^2 = 2s \sin^2(\theta/2)$ or $|\psi|^2 = \tan^2(\theta/2)$, θ being the deviation angle of the classical spin $\langle S \rangle$ from the $0Z$ axis. These parametrisations are evidently coupled by the formula

$$|\psi|^2 = |\alpha|^2/2s(1 - |\alpha|^2/2s)^{-1}$$

which naturally contains a singular point, $|\alpha|^2 = 2s$, according to their geometric images, namely SCS are defined by the points of the sphere $\mathbb{S}^2$ and Glauber CS by those of the complex plane $\mathbb{C}$.

So at $s = \infty$ the choice of the procedure to be applied is a matter of taste.

(ii) When 'quantum corrections' of the order of $1/s$ or higher are to be studied in the first approach, the HP series *must be truncated* because this series is asymptotic. In fact, as a result of the operators ordering, the coefficients C_m of all the terms $(a^+)^m(a)^n$ are asymptotic series in $1/s$:

$$C_m = \sum_{n=0}^{\infty} \alpha_{mn}(1/s)^n.$$

For example, one can estimate the magnitude of α_{1n} as

$$\alpha_{1n} \simeq (1/s\sqrt{2})(n/2es)^{n-2}$$

for $n \gg 1$ (here n is determined by the power of a non-ordered term $(\alpha^+\alpha)^n$ in the expansion of the HP square roots). This means that one can derive quantum corrections for finite s by truncating the series at the corresponding term, naturally depending on $1/s$; hence in our example $n < 2es$.

In [2] the result of the application of the truncated HP transformations was shown to depend centrally on the direction of the transformation quantisation axis, $0Z$, in the case of the anisotropic Hamiltonian; for example, an incorrect choice of this direction can lead not only to an incorrect magnon dispersion, but also to a 'damaged' ground state of the easy-plane Hamiltonian (see equation (15), case (2) in [2]) where quantum corrections appear that are absent for the correct $0Z$ axis direction. The corollary of the geometric analysis of [2] may be qualitatively given as the following rule: *the HP transformation quantisation axis (0Z) should be directed along the 'easiest' axis of the Hamiltonian.*

(iii) Finally we note that, using the identity

$$(|\beta|_x^2)^2 = 2|\beta|^2\,|\beta_x|^2 + (\bar{\beta}^2\beta_x^2 + \bar{\beta}_x^2\beta^2).$$

Hamiltonian (6) of [2] exactly coincides with (11*b*) of [1] up to terms of the order of $|\alpha|^6$; that is, in both cases

$$\mathcal{H} = s^2\alpha J\{[2(\delta/\alpha^2)\,|\beta|^2 + |\beta_x|^2](1 - |\beta|^2/2) + \tfrac{1}{4}(|\beta|_x^2)^2(1 + 2\delta + |\beta|^2/4)\}^2.$$

Ultimately, one can conclude that the use of the HP transformation in the limit $s \to \infty$ carries no extra information compared to that contained in the SU(2) averaged (Landau–Lifshitz) Hamiltonian which is well studied.

References

[1] Škrinjar M J, Kapor D V and Stojanović S D 1989 *J. Phys.: Condens. Matter* **1** 725–32
[2] Makhankov A and Makhankov V 1988 *Phys. Status Solidi* b **145** 669–78